

# Testing Groups of Genes

Manuela Hummel, LMU München

Adrian Alexa, MPI Saarbrücken

Ulrich Mansmann, LMU München

NGFN-Courses in Practical DNA Microarray Analysis

Heidelberg, March 6, 2008

# Overview

---

## Analysis of groups of genes

- Motivation
- How to define gene groups
- Assess relevance of gene groups

## Group testing methods

- **Gene set enrichment:** Fisher-test, GSEA
- **Holistic approaches:** Category, globaltest, GlobalAncova

# Motivation

---

## So far: Gene-wise analysis

- Genes are treated independently
- Correction for multiple testing is crucial
- Resulting lists of interesting genes are rather 'unstable'
- Biological interpretation of such gene lists is hard

## Now: Analysis of gene sets

- Predefined gene groups provide more biological knowledge
- More meaningful interpretation in biological context
- Number of gene sets to be investigated is smaller than number of individual genes
- Useful for validation of published gene groups  
*Example: Does a gene signature have predictive value?*

# How to Define Gene Groups

---

Exploratory research, literature search or Bioinformatic algorithms can be used to define

- Pathways  
Networks of interacting genes (KEGG, cMAP, BioCarta)
- Gene Ontology categories  
Biological Process, Molecular Function, Cellular Component
- Regions in the genome
- Signatures for classification
- Gene sets of published results
- ...

# Assess Relevance of Gene Groups

---

- Outstanding gene expression in a specific group compared to other genes

*Example: Do the cyclin D1 target genes show an extraordinary expression pattern in human tumours?*

- Differential gene expression not of single genes but over a specific group of genes

*Example: Does the cell cycle pathway contain (many) differentially expressed genes between cancer types A and B?*

- Two basic strategies for analysis:

Gene set enrichment and holistic approaches

# Group Testing

---

## Gene set enrichment

- **Idea:** Provide biological meaning to a list of interesting genes by means of an over-representation analysis
- **Step 1:** Gene-wise analysis (e.g. of differential expression)  
**Step 2:** Score gene groups for enrichment  
(always in comparison with the set of all genes)
- **Goal:** Find gene groups that contain **many interesting genes**

## Holistic approaches

- **Idea:** Look directly at gene sets and ask whether they are biologically relevant with respect to differential expression
- Global analysis of differential expression for gene groups  
(without taking the set of all genes as a reference)
- **Goal:** Find gene groups that contain **at least one interesting gene** or **many genes with moderate differentiability**

# Hypergeometric Test

---

## Step 1

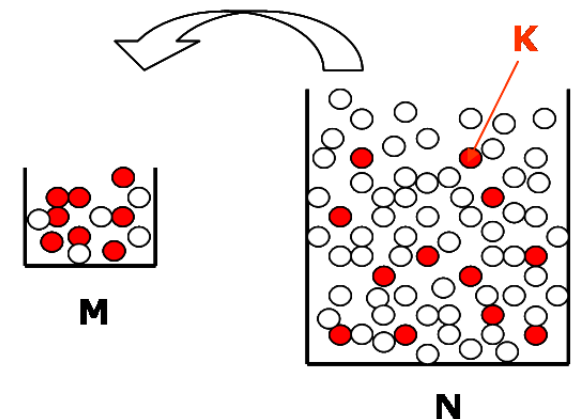
- Compute a gene-wise measure (for differential expression, e.g. t-statistic p-values)
- Adjust for multiple testing and choose a cutoff to **define a list of interesting genes**

## Step 2

- Given  $N$  genes on the microarray and  $M$  genes in a gene group, what is the probability of **having  $x$  from  $K$  interesting genes in this group?**

$$P(X = x|N, M, K) = \frac{\binom{M}{x} \binom{N-M}{K-x}}{\binom{N}{K}}$$

- A p-value for the gene group corresponds to  $P(X \geq x|N, M, K)$



# Fisher's Exact Test

---

- The hypergeometric test is equivalent to Fisher's exact test

	∈ gene group	∉ gene group	
∈ DE genes	$x$	$K - x$	$K$
∉ DE genes	$M - x$	$(N - M) - (K - x)$	$N - K$
	$M$	$N - M$	$N$

- Fisher-test and similar tests based on gene counts are very often used in Gene Ontology analysis  
(binomial test,  $\chi^2$  test, test based on normal z scores)  
*Khatri and Draghici (2005)*
- All these tests have the hypergeometric as null distribution  
*Rivals et al. (2006)*



# Fisher's Exact Test

---

**Example:**  $N = 20000$  genes on the microarray,  $M = 100$  genes in a gene group of interest,  $K = 300$  differentially expressed genes

	∈ group	∉ group	
∈ DE	3	297	300
∉ DE	97	19603	19700
	100	19900	20000

could be random

p-value = 0.19

	∈ group	∉ group	
∈ DE	6	294	300
∉ DE	94	19606	19700
	100	19900	20000

not likely random

p-value = 0.004

# Fisher's Exact Test

---

## Advantages

- Not restricted to analysis of differential expression
- If you just get a list of somehow interesting genes and want to assess biological background, tests based on gene counts are the only way to go

## Problems

- Loss of information because of two separated steps
- Small but consistent differential expression is not detected
- Dividing genes into differentially and non-differentially expressed genes is artificial
- No clear way of defining  $K$ : p-value correction and choice of a cutoff are crucial

# Gene Set Enrichment Analysis

---

*Subramanian et al. (2005)*

## Step 1

- Compute a gene-wise measure (for differential expression, e.g. absolute t-statistics)
- Rank genes according to this measure

## Step 2

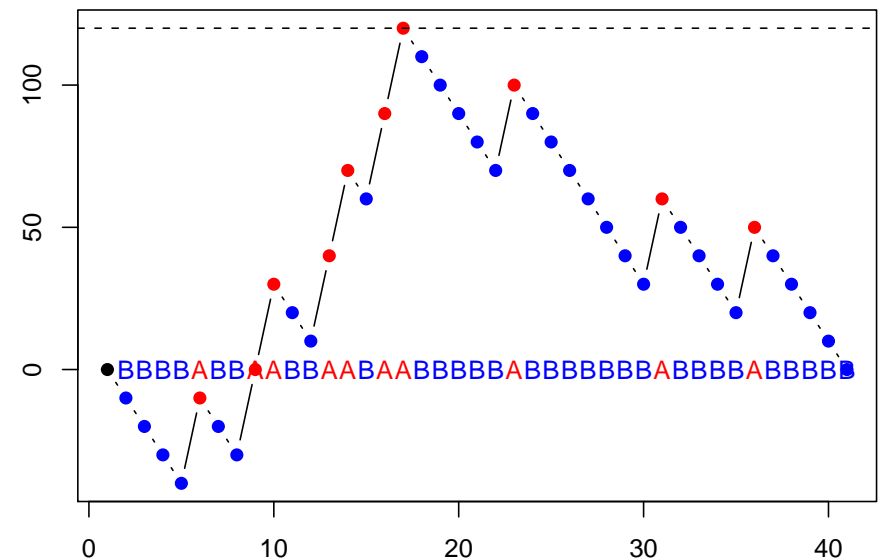
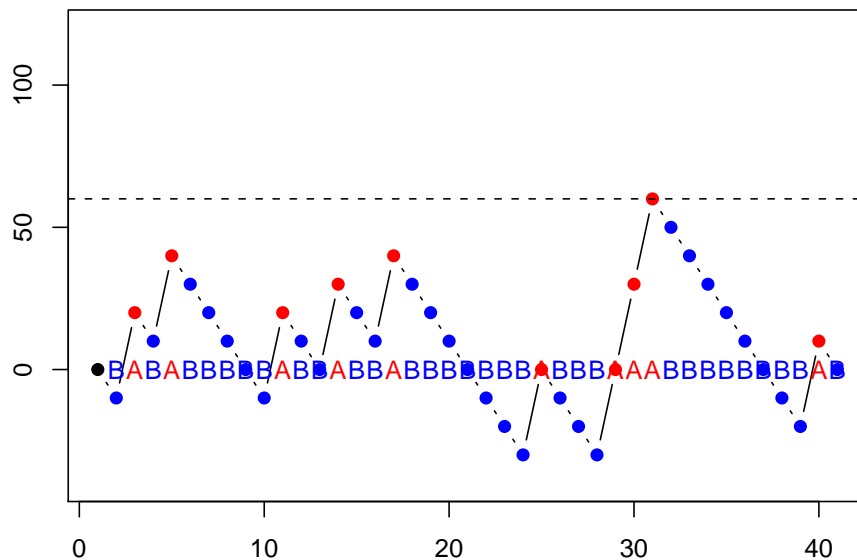
- Assign labels **A** to genes belonging to a gene group of interest and **B** to all the other genes
- If group A is enriched with interesting genes, many of it's genes will have high ranks and we will observe a separation in the ordered list

A B A A B A A A B A B B B A B B B B A B B B

measure for differential expression

# Gene Set Enrichment Analysis

- Assign score  $n_B$  to all genes **A** and  $-n_A$  to all genes **B**
- Draw the cumulative sum of these scores
- Is the maximum  $M$  of the cumulative sum unusually high? (Kolmogorov-Smirnov test)



# GSEA Permutation Test

---

## Permute genes

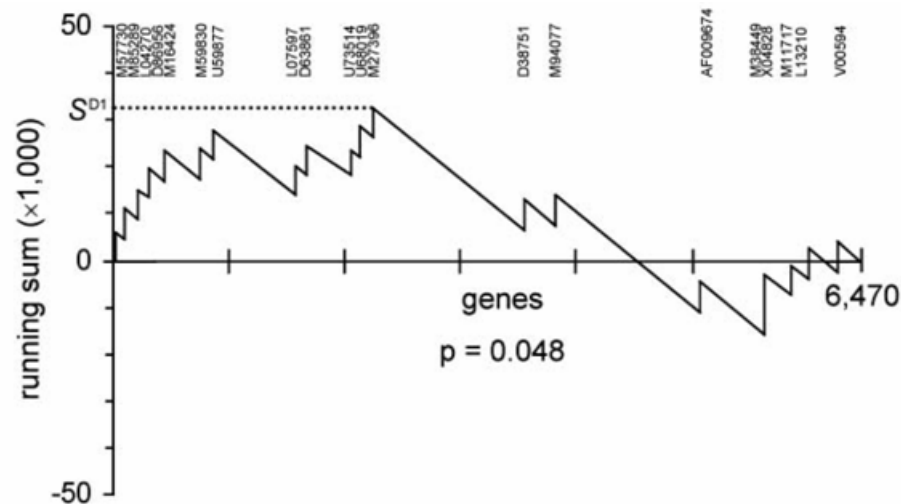
- Permute labels **A** and **B** in the ordered list  $P$  times
- Calculate the maximum  $M^*$  of the cumulative sum for each permutation
- Empirical p-value:  $p = \#(M^* \geq M) / P$
- Hypothesis: group is extreme w.r.t. random mixing

## Permute subjects

- Permute phenotype labels in the expression matrix
- Compute the gene-wise measure for each permutation
- For each resulting gene ranking calculate  $M^*$  and then a p-value as above
- Hypothesis: group is extreme w.r.t. overall expression

# GSEA Example

- *Lamb et al. (2003)* investigate activity of cyclin D1 in human tumours: Does the cyclin D1 target gene set play a prominent role in different tumour entities? Being present as highly expressed genes
- **Group A:** cyclin D1 target gene set  
**Group B:** all other genes



# Gene Set Enrichment Analysis

---

## Advantages

- Not restricted to analysis of differential expression
- Ranking of genes is considered
- No cutoff has to be chosen

## Problems

- Loss of information because of two separated steps
- Small but consistent differential expression is not detected

# Category

---

*Gentleman (2006)*

- Goal is to find gene categories whose genes show small but consistent **expression changes in the same direction**
- Calculate vector  $\mathbf{x}$  of genewise statistics indicating differential expression, e.g. t-test statistics or more general  $\mathbf{x} = f_1(\mathbf{X})$
- Get an incidence matrix  $\mathbf{A}$  representing the mappings between predefined categories and genes

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & \dots \\ 0 & 0 & 0 & 1 & 0 & \dots \\ 1 & 1 & 0 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{array}{l} \leftarrow \text{categories} \\ \\ \uparrow \\ \text{genes} \end{array}$$

- Row sums: numbers of genes in each category  
Column sums: numbers of categories each gene belongs to



# Category

---

- Define a statistic  $\mathbf{z}$  that reflects which categories are extreme:

$$\mathbf{z} = \frac{\mathbf{Ax}}{\sqrt{\text{rowsums}(\mathbf{A})}} \quad \text{or more general} \quad \mathbf{z} = f_2(\mathbf{A}, \mathbf{x})$$

- When  $\mathbf{x}$  is a vector of t-statistics and  $\mathbf{z}$  as shown, then  $\mathbf{z} \sim N(0, 1)$  (unfortunately only when genes are independent)
- Comparisons are possible
  - Within categories:** For a given category, is the observed test statistic unusual?
  - Between categories:** Are any of the observed category statistics unusually w.r.t. the entire reference distribution?

# Category Permutation Test

---

## Permute genes

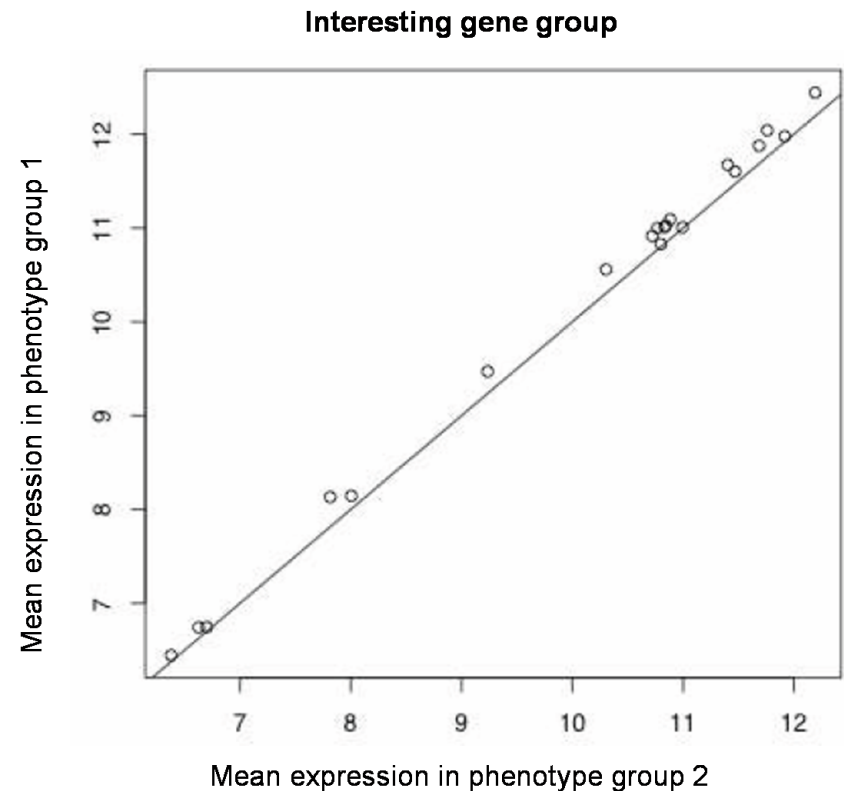
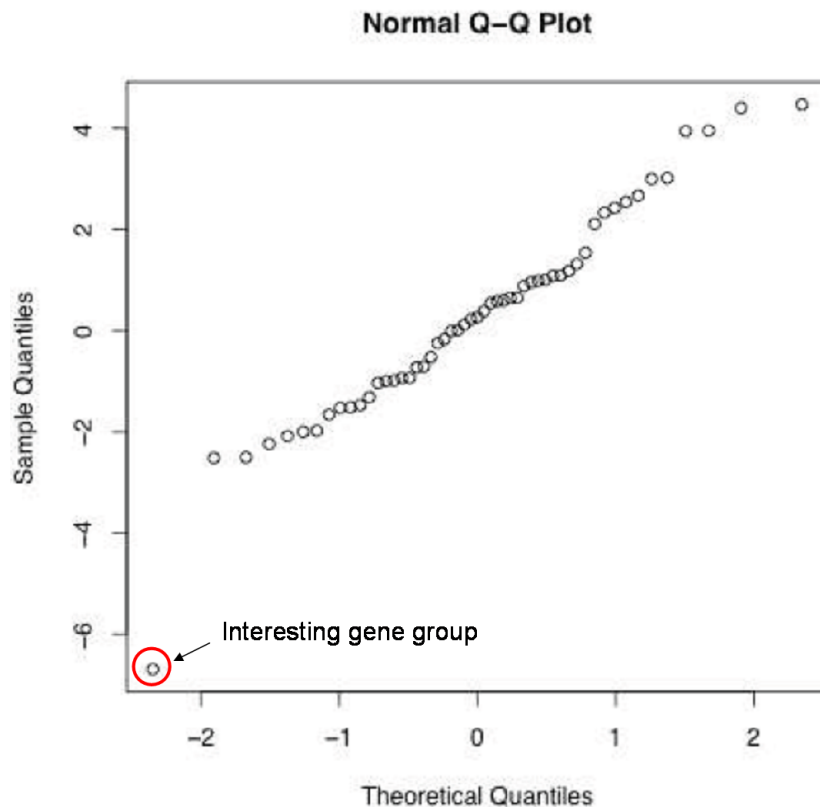
- Permute columns of  $\mathbf{A}$   $P$  times
- Calculate category statistic  $\mathbf{z}^*$  for each permutation
- Empirical p-value:  $p = \#(\mathbf{z}^* \geq \mathbf{z})/P$
- Hypothesis: group is extreme w.r.t. random mixing

## Permute subjects

- Permute phenotype labels in the expression matrix  $\mathbf{X}$
- Compute the gene-wise measure  $\mathbf{x}^*$  for each permutation
- Calculate category statistic  $\mathbf{z}^*$  with  $\mathbf{A}$  and each  $\mathbf{x}^*$  and then a p-value as above
- Hypothesis: group is extreme w.r.t. overall expression

# Category

- qq-plots of the category statistics can help to reveal interesting gene groups
- These groups can further be explored by plotting expression means in the two clinical entities against each other



# Category

---

## Advantages

- Proper statistical framework
- Very flexible through choice of functions  $f_1$  and  $f_2$
- Ability to find groups with interesting expression patterns missed by gene set enrichment approaches

## Problems

- Categories with both up- *and* down-regulated genes will eventually not be found because their t-statistics will cancel out in the overall sum
- Permutation of genes destroys correlations between genes – permutation of subjects seems more reasonable

# Global Tests

---

Is the global expression pattern of a group of genes significantly related to some clinical variable of interest?

**globaltest**: Does knowledge of gene expression  $X$  help to improve prediction of the variable  $Y$ ?

$$H_0 : P(Y = 1|X) = P(Y = 0|X)$$

*Goeman et al. (2004)*

**GlobalAncova**: How is gene expression  $X$  influenced by the structure of the variable  $Y$ ?

$$H_0 : P(X|Y = 1) = P(X|Y = 0)$$

*Mansmann & Meister (2005), Hummel et al. (2008)*

Tests are equivalent under the null hypothesis of no relationship between  $Y$  and  $X$

# Globaltest

---

- Does knowledge of gene expression  $X$  help to improve prediction of the variable  $Y$ ?

- Test statistic

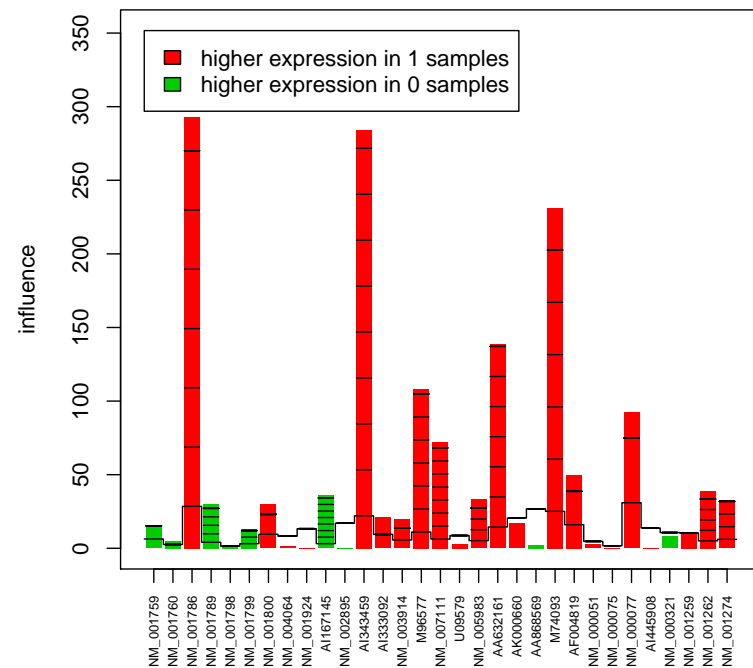
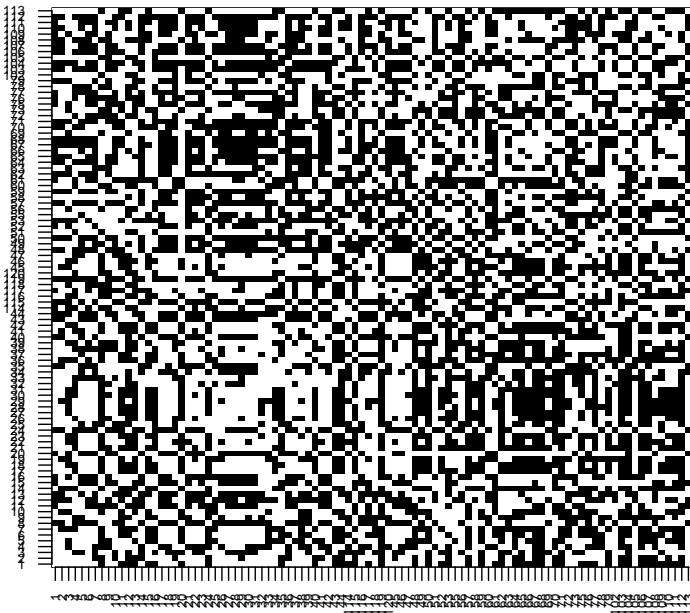
$$\begin{aligned} Q &\sim (Y - \mu)^T R (Y - \mu) \\ &\sim \sum_g [X_g (Y - \mu)]^2 && \text{sum over genes} \\ &\sim \sum_i \sum_j R_{ij} (Y_i - \mu)(Y_j - \mu) && \text{sum over subjects} \end{aligned}$$

$R = X^T X$  matrix of correlations between gene expression of subjects

- Test to see whether **subjects with similar expression also have similar outcomes**
- Permutation based and asymptotic p-values are available
- Also multicategorical, continuous or survival variables can be considered and adjustment for covariates is possible

# Globaltest

- Checkerboard plots help to illustrate whether subjects of the same clinical group also have similar expression patterns
- Gene plots show the influence of single genes in the gene sets on the global test statistic



# GlobalAncova

---

- How is gene expression  $X$  influenced by the structure of the variable  $Y$ ?
- The expectation for gene  $j$  follows a linear model  $E(x_j) = D\beta_j$
- The design matrix  $D$ , e.g. in the two group case and with an additional covariate  $z$ , may look like this

$$\begin{array}{l} \text{sample 1} \\ \text{sample 2} \\ \text{sample 3} \\ \text{sample 4} \\ \dots \end{array} \begin{array}{c} \text{Int} \quad Y \quad z \\ \left( \begin{array}{ccc} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ \dots & \dots & \dots \end{array} \right) \end{array}$$

- The full model containing the clinical parameter of interest is compared to a reduced model without it via the [extra sum of squares principle](#)
- Gene-wise linear models are summarized to a [global F-test](#)



# GlobalAncova

---

- **Permutation p-values:**

Permutation of subjects and calculation of empirical p-values

- **Asymptotic p-values:**

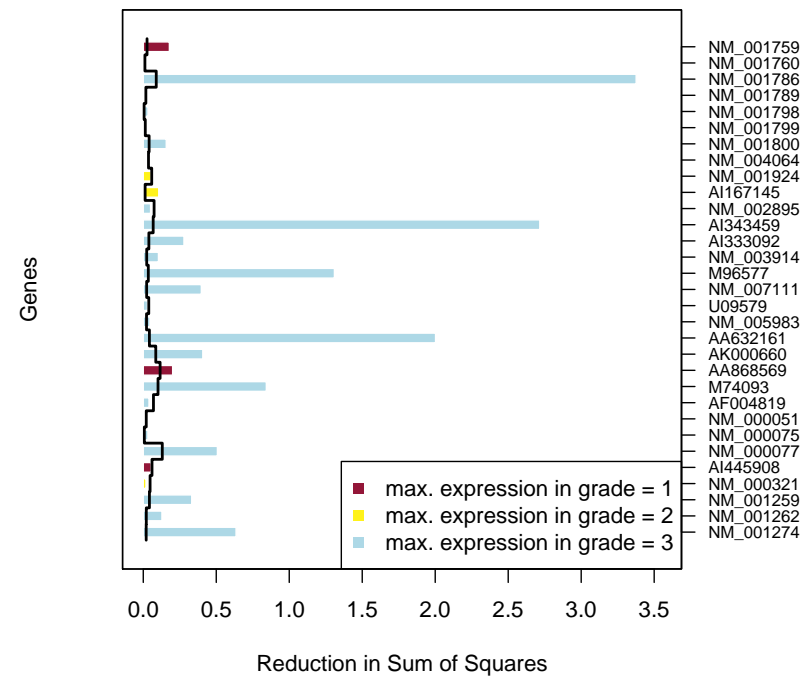
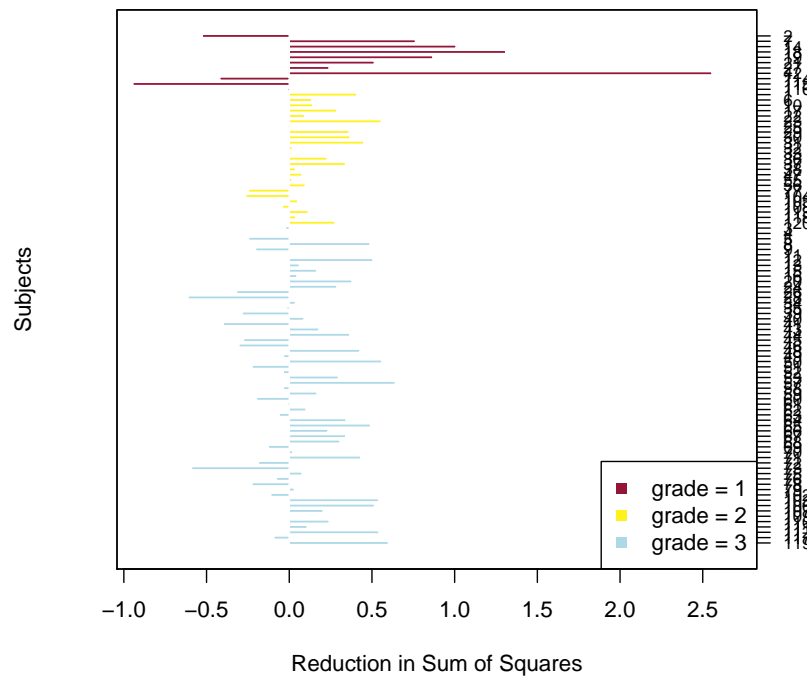
Approximation of the test statistic distribution

- **General linear model framework** allows analysis of

Design	Full model	Reduced model
Various groups	$\sim \text{group} + \text{cov}$	$\sim \text{cov}$
Dose-response	$\sim \text{dose} + \text{cov}$	$\sim \text{cov}$
Group by dose interaction	$\sim \text{group} * \text{dose} + \text{cov}$	$\sim \text{group} + \text{dose} + \text{cov}$
Differential time trends	$\sim \text{group} * \text{time} + \text{cov}$	$\sim \text{group} + \text{time} + \text{cov}$
Gene gene interaction	$\sim \text{gene} + \text{cov}$	$\sim \text{cov}$
Differential co-expression	$\sim \text{group} * \text{gene} + \text{cov}$	$\sim \text{group} + \text{gene} + \text{cov}$
...		

# GlobalAncova

- Subject plots help to detect subjects that 'do not fit' into their clinical groups
- Gene plots show the influence of single genes in the gene sets on the global test statistic



# Global Tests

---

## Advantages

- Gene groups with few strongly as well as groups with many moderately differentially expressed genes are detected
- Flexible frameworks suitable for many kinds of applications

## Problems

- Only analysis of expression patterns within groups – it is not accounted for the overall distribution of group statistics
- Eventually too sensitive for data with much differential expression

# Gene versus Subject Sampling

---

*Goeman and Bühlmann (2007)*

**Subject sampling model:** A new sample corresponds to measurements of the same variables (= genes) for a new subject

**Gene sampling model:** A new sample would correspond to a sample of new genes for the same subjects  
(this is also the underlying model for hypergeometric tests)

- Gene sampling **reverses the roles of samples and variables**
- Interpretation of p-values is different
- **Misleading sample size** in gene sampling model, i.e. the number of genes  $m$  does not correspond to the biological sample size  $n = \text{number of subjects}$
- **Assumption of independence between genes** in the gene sampling model may lead to anti-conservative tests

# Summary: Two Perspectives on Gene Groups

---

## Question 1

Is the gene expression in gene set A different from the expression in gene set B?

**Gene set A**

**Gene set B**

## Question 2

Is there differential expression between different biological entities, not in terms of single genes but with respect to a defined gene set?

**Entity 1**

**Gene set X**

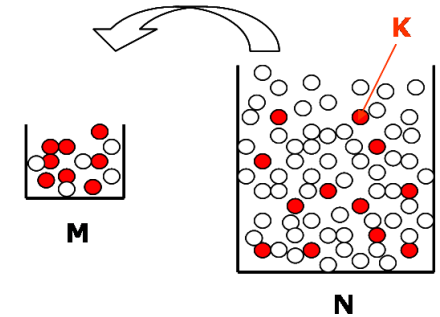
**Entity 2**

**Gene set X**

# Summary: Perspectives of Group Testing

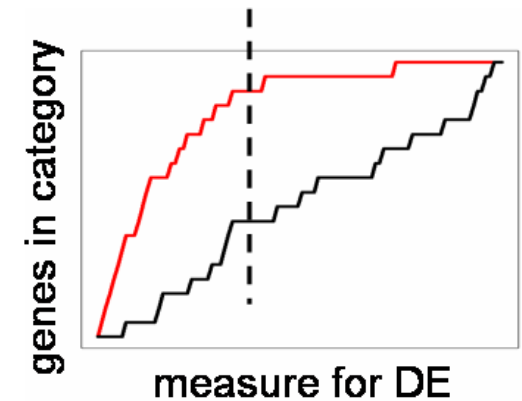
## Fisher-test approaches

Are there more interesting genes in the gene set than expected by randomly drawing?



## Gene set enrichment analysis

Do the genes in the gene set have high ranks with respect to differential expression?



## Globaltest / GlobalAncova / Category

Can there be found differential expression in the gene set?



# Outlook

---

- **Gene versus subject sampling**: Also tests based on gene counts in a contingency table could be modified to subject sampling procedures
- **Annotation**: Only genes annotated to the considered gene sets are involved in the analysis, all others are missed
- When testing large collections of gene sets we have to face a **multiple testing problem**
- **Dependencies between gene sets** complicate statistical analysis and interpretation  
Special example: Gene Ontology

# References

---

1. Gentleman R with contributions from Falcon S. Category: Category Analysis. R package version 2.0.0.
2. Goeman JJ, de Kort F, van de Geer SA, van Houwelingen JC. A global test for groups of genes: testing association with a clinical outcome. *Bioinformatics* 2004; 20 (1): 93-99.
3. Goeman JJ, Bühlmann P. Methodological issues in gene set testing based on microarray data. *Bioinformatics* 2007; 23 (8): 980-987.
4. Hummel M, Meister R and Mansmann U. GlobalANCOVA: exploration and assessment of gene group effects. *Bioinformatics* 2008; 24 (1): 78-85.
5. Khatri P, Draghici S. Ontological analysis of gene expression data: current tools, limitations, and open problems. *Bioinformatics* 2005; 21 (18): 3587-95.
6. Lamb J, Ramaswamy S, Ford HL, Contreras B, Martinez RV, Kittrell FS, Zahnow CA, Patterson N, Golub TR, Ewen ME. A mechanism of Cyclin D1 Action Encoded in the Patterns of Gene Expression in Human Cancer. *Cell* 2003; 114: 323-334.
7. Mansmann U, Meister R. Testing differential gene expression in functional groups. *Methods Inf Med* 2005; 44 (3).
8. Rivals I, Personnaz L, Taing L, Potier MC. Enrichment or depletion of a GO category within a class of genes: which test? *Bioinformatics* 2007; 23 (4): 401-407.
9. Subramanian A, Tamayo P, Mootha VK, Mukherjee S, Ebert BL, Gillette MA, Paulovich A, Pomeroy SL, Golub TR, Lander ES, Mesirov JP. Gene set enrichment analysis: a knowledge-based approach for interpreting genome-wide expression profiles. *PNAS* 2005; 102 (43): 15545-15550.